

Zemach and magnetic radius of the proton from the hyperfine splitting in hydrogen

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Abstract. The current status of the determination of corrections to the hyperfine splitting of the ground state in hydrogen is considered. Improved calculations are provided taking into account the most recent value for the proton charge radius. Comparing experimental data with predictions for the hyperfine splitting, the Zemach radius of the proton is deduced to be 1.045(16) fm. Employing exponential parametrizations for the electromagnetic form factors we determine the magnetic radius of the proton to be 0.778(29) fm. Both values are compared with the corresponding ones derived from the data obtained in electron-proton scattering experiments and the data extracted from a rescaled difference between the hyperfine splittings in hydrogen and muonium.

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1 Introduction

High-precision measurements and calculations of energy spectra of hydrogen-like atoms provide tests of quantum electrodynamics (QED) with very high precision (see [1–4] and references therein). In some cases, the current accuracy of QED calculations exceeds those of the known values of fundamental physical constants. For instance, recent measurements and calculations of the g factor of hydrogen-like carbon and oxygen have provided the basis for a new determination of the electron mass (see [5] and references therein). Measurements of the Lamb shift in hydrogen, combined with corresponding calculations, have facilitated to determine the Rydberg constant and to deduce an improved value for the proton charge radius [6–9].

The relative experimental accuracy of the ground-state hyperfine splitting in hydrogen is better than 10^{-12} [10]. The error associated with the QED corrections to the hyperfine splitting is estimated to contribute on the level 10^{-9} . The major theoretical uncertainty arises from nuclear structure-dependent contributions. The most important structure-dependent term is the proton-size correction, which is determined exclusively by the spatial distributions of the charge and the magnetic moment of the proton. It contributes on the relative level 10^{-5} . Assuming that all other theoretical corrections are accurately known, one can determine the proton-size contri-

bution by comparing theoretical and experimental values for the hyperfine splitting in hydrogen. The major goal of the present paper is to determine the Zemach and the magnetic radius of the proton by such a comparison.

In Section 2, we consider various theoretical contributions to the hyperfine splitting and derive the proton-size correction comparing theory and experiment. In Section 3, we refine the value of the proton-size correction by recalculating some of these contributions and determine the magnetic radius of the proton, employing an exponential parametrization for the electric and magnetic form factors. The recalculation of the recoil correction has improved the value of the Zemach radius compared to the previous result obtained in [11]. In Section 4, the results obtained are compared with corresponding data derived from elastic electron-proton scattering experiments [12–14] and the data extracted from a rescaled difference between the hyperfine splittings in hydrogen and muonium [15].

The relativistic units $\hbar = c = 1$ are used throughout the paper.

2 Hyperfine splitting in hydrogen

The hyperfine splitting of the ground state in hydrogen can be written in the form

$$\Delta E_{\text{theor}} = E_F (1 + \delta^{\text{Dirac}} + \delta^{\text{QED}} + \delta^{\text{structure}}), \quad (1)$$

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Table 1. Numerical values for various corrections to the hyperfine splitting in hydrogen together with the assigned errors. The energies ΔE_{exp} and E_{F} are given in units of MHz.

	Value	Error	Ref.
ΔE_{exp}	1 420.405 751 767	0.000 000 001	[10]
E_{F}	1 418.840 08	0.000 02	[6]
$\Delta E_{\text{exp}}/E_{\text{F}}$	1.001 103 49	0.000 000 01	
δ^{Dirac}	0.000 079 88		[17]
δ^{QED}	0.001 056 21	0.000 000 001	[18–23]
δ^{ps}	−0.000 040 11	0.000 000 61	
δ^{recoil}	0.000 005 97	0.000 000 06	[25,31], this work
δ^{pol}	0.000 001 4	0.000 000 6	[24]
$\delta^{\mu\text{vp}}$	0.000 000 07	0.000 000 02	[25]
δ^{hvp}	0.000 000 01		[26,27]
δ^{weak}	0.000 000 06		[28,29]

where E_{F} is the Fermi energy [16]

$$E_{\text{F}} = \frac{8}{3}\alpha(\alpha Z)^3 \frac{m_e^2 m_p^2}{(m_e + m_p)^3} \frac{\mu_{\text{p}}}{\mu_{\text{N}}}, \quad (2)$$

μ_{p} is the magnetic dipole moment of the proton, μ_{N} is the nuclear magneton, m_e and m_p are the electron and proton mass, respectively. The relativistic correction δ^{Dirac} can easily be obtained from the Dirac equation [17]:

$$\delta^{\text{Dirac}} = \frac{3}{2}(\alpha Z)^2 + \frac{17}{8}(\alpha Z)^4 + \dots \quad (3)$$

Here and in what follows we keep the nuclear charge number Z to separate the relativistic and radiative corrections. For recent achievements in calculations of the radiative correction δ^{QED} we refer to references [18–23]. The uncertainty of δ^{QED} is mainly determined by uncalculated terms of order $\alpha^3(\alpha Z)$ and by uncertainties associated with some of the calculated terms. The structure-dependent correction $\delta^{\text{structure}}$ is usually expressed as the sum

$$\delta^{\text{structure}} = \delta^{\text{pol}} + \delta^{\mu\text{vp}} + \delta^{\text{hvp}} + \delta^{\text{weak}} + \delta^{\text{rigid}}. \quad (4)$$

The part associated with intrinsic proton dynamics (polarizability) δ^{pol} has been recently evaluated in [24] employing experimental and theoretical results for the structure functions of polarized protons. The correction due to muonic vacuum-polarization $\delta^{\mu\text{vp}}$ has been obtained in [25], while the hadronic vacuum-polarization contribution δ^{hvp} was evaluated in [26,27]. For the weak interaction term δ^{weak} we refer to references [28,29]. Values for all these corrections together with corresponding uncertainties are presented in Table 1. The Fermi energy E_{F} is evaluated employing the values of the fundamental constants tabulated in [6]. The leading chiral logarithms contributions to the structure-dependent correction have been also investigated within an effective field theory [30].

Now let us turn to the term δ^{rigid} , which is determined by electric and magnetic form factors of the proton. This quantity can be decomposed into two parts: $\delta^{\text{rigid}} = \delta^{\text{ps}} + \delta^{\text{recoil}}$, where δ^{ps} represents the proton-size correction and δ^{recoil} is associated with recoil effects. The

recoil part contains both terms arising from a pointlike Dirac proton and additional recoil correction due to the internal proton structure. Following reference [31] we do not separate them. Calculations of the dominant contribution (relative order $(\alpha Z)m_e/m_p$) to the recoil correction have a long history (see [31] and references therein). The contribution of the order $(\alpha Z)^2 m_e/m_p$ has been first derived in [31], while the radiative-recoil correction of the order $\alpha(\alpha Z)m_e/m_p$ has been obtained in [25]. To determine the magnetic radius of the proton from the hydrogen hyperfine splitting we propose the following. At first we calculate the structure-dependent part of the recoil correction in a rough approach, taking the proton magnetic radius to be the same as the charge one. Then we find the proton-size correction from a comparison of the experimental and theoretical values of the hyperfine splitting. Using the dipole parametrizations of the form factors we extract a preliminary value for the proton magnetic radius. Then we recalculate the recoil-structure correction with the obtained value of the proton magnetic radius and take into account the radiative and binding contributions to the proton-size term. At last we again find the proton-size correction and extract the magnetic radius.

At first iteration we have calculated the recoil-structure correction (integrals VO, VV, κ^2 , No. 1, and No. 2 of Ref. [31]) with the new proton charge radius $\langle r_{\text{E}}^2 \rangle^{1/2} = 0.8750(68)$ fm [6] and with the same value for the magnetic radius of the proton. The total recoil correction turns out to be 5.84 ppm. Accordingly, the proton-size correction δ^{ps} is given by

$$\delta^{\text{ps}} = \Delta E_{\text{exp}}/E_{\text{F}} - 1 - \delta^{\text{Dirac}} - \delta^{\text{QED}} - \delta^{\text{recoil}} - \delta^{\text{pol}} - \delta^{\mu\text{vp}} - \delta^{\text{hvp}} - \delta^{\text{weak}}. \quad (5)$$

This equation yields the value $\delta^{\text{ps}} = -39.98(61) \times 10^{-6}$. The next section is devoted to the deduction of the magnetic radius of the proton from the proton-size correction and to the next iteration.

3 Magnetic radius of the proton

The proton-size correction has been first evaluated in the non-relativistic limit by Zemach [32]:

$$\delta^{\text{Zemach}} = -2\alpha Z \frac{m_e m_p}{m_e + m_p} R_p, \quad (6)$$

where

$$\begin{aligned} R_p &= \int d^3r d^3r' \rho_E(\mathbf{r}) \rho_M(\mathbf{r}') |\mathbf{r} - \mathbf{r}'| \\ &= -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[\frac{\mu_N}{\mu_p} G_E(Q^2) G_M(Q^2) - 1 \right] \end{aligned} \quad (7)$$

defines the Zemach radius of the proton. Here $\rho_E(\mathbf{r})$ and $\rho_M(\mathbf{r})$ denote the nuclear charge and magnetization distribution, respectively, where both densities are normalized to unity. The quantities $G_{E/M}$ represent the electric and magnetic form factors, respectively. Since we deal here only with the static limit ($Q^0 = 0$), we define them to be dependent only on the spatial momentum transfer (squared), $Q^2 > 0$. The charge and magnetic mean-square radii are defined by the formula

$$\langle r^2 \rangle_{E/M} = \int d^3r r^2 \rho_{E/M}(\mathbf{r}). \quad (8)$$

In a first step we can approximate the proton-size correction by the Zemach formula. Then one can easily find the Zemach radius $R_p = 1.058$ fm. The usual experimental fit for the proton form factors is the dipole parametrization

$$G_D(Q^2) = \frac{1}{[1 + Q^2 R_D^2]^2}, \quad (9)$$

which comes from the exponential model of the charge/magnetization distribution. But recently, it was obtained in Jefferson Laboratory (JLab) [33–36], that for $Q \geq 1$ GeV the behavior of the electric form factor differs from the dipole parametrization. However, the Zemach correction is not sensitive to the form factors behavior for $Q > 0.8$ GeV. As it was shown in reference [15], the contribution to the Zemach correction from the region $Q > 0.8$ GeV is the same for different experimental models of the proton electric and magnetic form factors. Therefore, in what follows, we use the dipole parametrizations for the form factors. The parameters R_D , one for the electric and another for the magnetic form factor, are directly connected with the corresponding values of the root-mean-square radii. For the charge radius of the proton we take the value $\langle r^2 \rangle_E^{1/2} = 0.8750(68)$ fm [6] obtained from the latest comparison of the theoretical and experimental values of the Lamb shift in hydrogen. Fixing the charge radius, we fit the magnetic radius such as to reproduce the Zemach radius. As the result we find a preliminary magnetic radius of the proton: $\langle r^2 \rangle_M^{1/2} = 0.800$ fm. In order to estimate the error associated with the model dependence we consider also the model

$$G_{\text{JLab}}(Q^2) = \left(1 - 0.13 \frac{Q^2}{\text{GeV}^2} \right) G_D(Q^2), \quad (10)$$

which is known as JLab model [34]. The error appeared is on the level of about 0.75%.

In a second step we account for corrections to the Zemach formula

$$\delta^{\text{ps}} = \delta^{\text{Zemach}} + \delta^{\text{radiative}} + \delta^{\text{relativistic}}, \quad (11)$$

where $\delta^{\text{radiative}}$ is the radiative structure-dependent correction obtained in [25] and $\delta^{\text{relativistic}}$ is the binding correction derived in [37]. The radiative correction has been derived assuming the exponential model with the same parameter R_D for both charge and magnetization distributions, i.e.

$$\delta^{\text{radiative}} = -\delta^{\text{Zemach}} \frac{\alpha}{3\pi} \left[4 \log(m_e R_D) + \frac{4111}{420} \right]. \quad (12)$$

The accuracy of this approximation is sufficient for our purpose. Calculating $\delta^{\text{radiative}}$ for different R_D , we obtain $\delta^{\text{radiative}} = 0.0153(2)\delta^{\text{Zemach}}$. The binding correction has been expressed in terms of electric and magnetic moments of the proton:

$$\begin{aligned} \delta^{\text{relativistic}} &= \delta^{\text{Zemach}} (\alpha Z)^2 \left[\frac{7}{4} - \gamma - \log(2\alpha Z) \right] \\ &\quad - 2(\alpha Z)^3 m_e \langle r \rangle_E \left(\frac{\langle r \log(m_e r) \rangle_E}{\langle r \rangle_E} - \frac{839}{750} \right) \\ &\quad - \frac{(\alpha Z)^3 m_e R_0}{5} \left(\frac{3\langle r^4 \rangle_M}{2R_0^4} - \frac{19\langle r^6 \rangle_M}{42R_0^6} + \frac{19\langle r^8 \rangle_M}{360R_0^8} - \frac{2}{825} \frac{\langle r^{10} \rangle_M}{R_0^{10}} \right) \\ &\quad - (\alpha Z)^3 m_e R_0 \left(\frac{\langle r^2 \rangle_M}{R_0^2} - \frac{1}{10} \frac{\langle r^4 \rangle_M}{R_0^4} \right) \left(\log(m_e R_0) + \frac{1}{30} \right), \end{aligned} \quad (13)$$

where γ is Euler's constant and $\langle r^n \rangle_{E/M} = \int d^3r r^n \rho_{E/M}(r)$. In part, this equation has been derived for the homogeneously charged sphere model for the proton charge distribution (with $R_0 = \sqrt{5/3} \langle r^2 \rangle_E^{1/2}$). Nevertheless, the error induced by using this formula in comparison with other models for the charge distribution does not exceed 5%. Employing the exponential model for the charge and magnetization distributions with electric and magnetic radii, $\langle r^2 \rangle_E^{1/2} = 0.875$ fm and $\langle r^2 \rangle_M^{1/2} = 0.800$ fm, respectively, we obtain $\delta^{\text{relativistic}} = 0.0002\delta^{\text{Zemach}} + 1.4 \times 10^{-8}$. Thus the proton-size correction takes the form

$$\delta^{\text{ps}} = 1.0154(2)\delta^{\text{Zemach}} + 1.4 \times 10^{-8}. \quad (14)$$

In addition, we need to correct the dominant term of the recoil contribution with the magnetic radius $\langle r^2 \rangle_M^{1/2} = 0.800$ fm. As a result, the recoil correction turns out to be 5.94(6) ppm.

Deducing again the proton-size correction by means of equation (5), we find the Zemach radius of the proton:

$$R_p = 1.045(16) \text{ fm}. \quad (15)$$

This value differs from the one obtained in [11], $R_p = 1.037(16)$ fm, mainly due to the recalculated recoil corrections with the new more precise charge and magnetic moment distributions.

4 Discussion

The value for the Zemach radius obtained above enables us to determine an improved magnetic radius of the proton:

$$\langle r^2 \rangle_M^{1/2} = 0.778(29) \text{ fm}. \quad (16)$$

The corresponding uncertainty is mainly due to errors associated with the polarizability effect as well as the uncertainty of the charge radius of the proton. In Table 1 we present the final values for the contributions to the hyperfine splitting in hydrogen. The value for δ^{ps} has been obtained by means of the experimental energy splitting, according to equation (5).

Another way to determine the proton magnetic radius is based on experimental data from elastic electron-proton scattering. Accordingly, Friar and Sick have recently determined the Zemach radius of the proton to be $R_p = 1.086(12)$ fm [13] and the proton charge radius $\langle r^2 \rangle_E^{1/2} = 0.895(18)$ fm [12]. Based on these values for R_p and $\langle r^2 \rangle_E^{1/2}$ and employing an exponential parametrization for both electric and magnetic form factors, we find the value of the proton magnetic radius $\langle r^2 \rangle_M^{1/2} = 0.824(27)$ fm. This value is in a good agreement with the recent experimental value of Sick — 0.855(35) fm presented in [14], and also the result of Hammer and Meißner 0.857 fm [14] is not far away.

Recently Brodsky et al. [15] proposed another method to extract the Zemach radius. They considered a rescaled difference between the hyperfine splittings in hydrogen and muonium

$$\frac{\Delta E_{\text{exp}}^p/E_F^p}{\Delta E_{\text{exp}}^\mu/E_F^\mu} = 1 + \delta^{\text{hfs}}, \quad (17)$$

where p and μ indicate quantities which refer to the proton and muon, respectively, $\Delta E_{\text{exp}}^\mu = 4\,463.302\,765(53)$ MHz [38], and E_F^μ is the Fermi energy for muonium. Employing recent values of the fundamental constants [6] they have obtained $\delta^{\text{hfs}} = 145.51$ ppm [15]. The ground state hyperfine splitting in muonium can be written as

$$\Delta E_{\text{theor}}^\mu = E_F^\mu (1 + \delta^{\text{Dirac}} + \delta^{\text{QED}} + \delta_\mu^{\text{recoil}} + \delta_\mu^{\text{hvp}} + \delta_\mu^{\text{weak}}). \quad (18)$$

The corrections δ^{Dirac} and δ^{QED} are the same as in the case of hydrogen, δ_μ^{hvp} and δ_μ^{weak} are the hadronic vacuum-polarization and the weak interaction contributions, respectively, and $\delta_\mu^{\text{recoil}}$ is the recoil term, which consists of relativistic and radiative parts. From this formula together

with equations (1) and (17) one can immediately derive the proton structure correction

$$\delta_p^{\text{structure}} = \delta^{\text{hfs}} + \delta_\mu^{\text{recoil}} + \delta_\mu^{\text{hvp}} + \delta_\mu^{\text{weak}} + \delta^{\text{hfs}}(\delta^{\text{Dirac}} + \delta^{\text{QED}} + \delta_\mu^{\text{recoil}} + \delta_\mu^{\text{hvp}} + \delta_\mu^{\text{weak}}). \quad (19)$$

If, following to reference [15], we take into account only the relativistic part of the recoil correction [20,31,39–41] and neglect the contributions δ_μ^{hvp} and δ_μ^{weak} , we obtain $\delta_p^{\text{structure}} = -31.8$ ppm. This yields the Zemach radius $R_p = 1.019(16)$ fm [15] that differs significantly from our result, $R_p = 1.045(16)$ fm. We have found, however, that the difference disappears if one includes the omitted terms. This is mainly due to the radiative-recoil correction evaluated in [20,39–44]. With this term included, the total recoil correction is determined as $\delta_\mu^{\text{recoil}} = -178.33$ ppm. The hadronic vacuum-polarization contribution obtained in [26,27] is $\delta_\mu^{\text{hvp}} = 0.05$ ppm, while the value of the correction due to Z^0 -boson exchange yields $\delta_\mu^{\text{weak}} = -0.01$ ppm [28,29]. Substituting these values into expression (19), we find $\delta_p^{\text{structure}} = -32.64$ ppm. Utilizing the values presented in Table 1, we obtain for the proton-size correction $\delta^{\text{ps}} = -40.15$ ppm. Then the Zemach radius can be easily determined with the result $R_p = 1.047(16)$ fm, which is very close to the value obtained in this work, $R_p = 1.045(16)$ fm.

As one can see, only a disagreement with the value for the Zemach radius and, therefore, with this for the magnetic radius, as is obtained from the electron-proton scattering experiments remains. At present we have no explanation for this deviation. One may hope, that a new determination of the proton charge radius via the Lamb shift experiment with muonic hydrogen, which is now in progress at PSI (Paul Scherrer Institute) [45], will elucidate the situation. From the theoretical point of view, an independent calculation of the proton polarizability effect would be also desirable.

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